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ABSTRACT

This collection of activities resulted from a visit by the Polish author to the Teachers' Center Project at Southern Illinois University at Edwardsville in 1980. The set of 30 activity sheets involve blocks with various types of patterns depicted. Directions for using each sheet and questions to be answered are then given. Elementary school children can do most of the activities, although the mathematical equations in the last portion of the booklet may be beyond their understanding level. (MNS)

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SEQUENCES

BY
JERZY CWIRKO-GODYCKI

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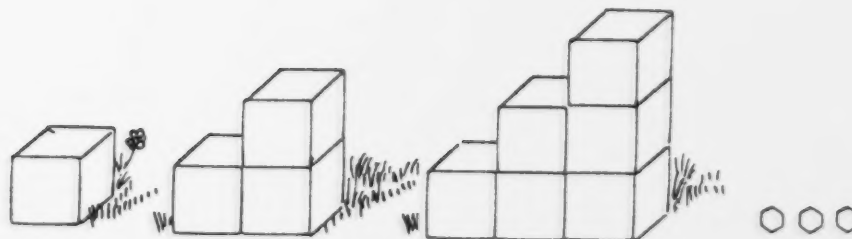
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Teachers' Center Project
Southern Illinois University at Edwardsville

SE 047 676

I N T R O D U C T I O N

It is with the greatest of pleasure that I welcome Dr. Jerzy Cwirko-Godycki's SEQUENCES to the growing collection of publications available to teachers through the Teachers' Center Project. This, as many readers will know, is the third of Jerzy's publications in our collection. It makes a major contribution to math education in saying, as do his other two books, that math is concerned with thinking and problem solving, not with the storage of fragmented and inert recipes. For those of you unacquainted with Jerzy's two previous books, Mathematical Activities from Poland I and II, we suggest a look-see. The three books make available some of the thinking of one of the most able scholars we've had the fortune to meet.

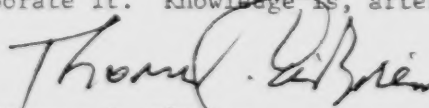
The problems in this book speak for themselves. A word about the end-of-book commentary is, however, appropriate. The commentary is brief and readers are invited to extend it. The purpose of the commentary was to help teachers to get started but Jerzy was quite deliberate in not providing an answer book or handbook. "Teachers are too able for me to do their thinking," he said.

It's not out of order, I hope, to say public Thanks to Jerzy, our Visiting Scholar in spring 1980, for his good will, good humor, and good sense. What a pleasant and fruitful time we had!

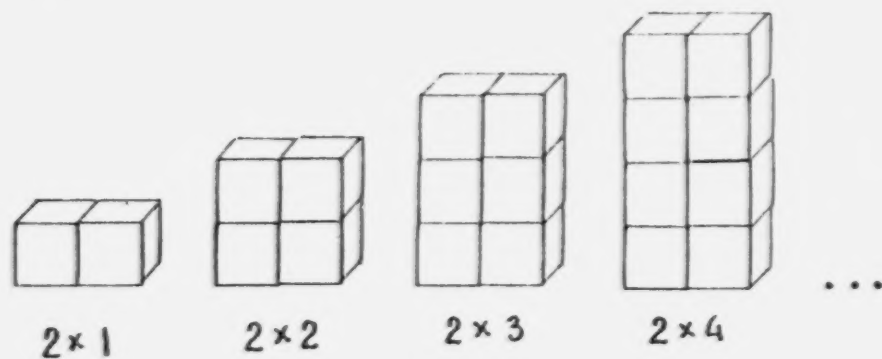
The present volume brings the number of Teachers' Center Project publications well into the sixties. If the spirit of Jerzy's work catches your fancy, inquire about the other publications we have available.

Best of all, if what's here interests you, extend it and elaborate it. Knowledge is, after all, not a spectator sport.

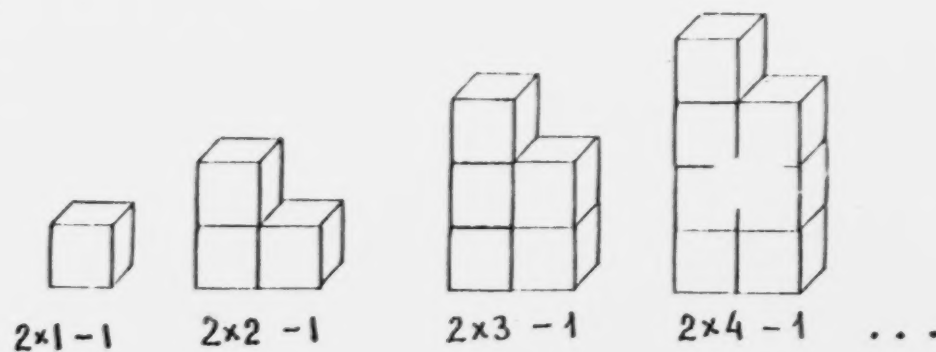
July 21, 1980


Thomas C. O'Brien

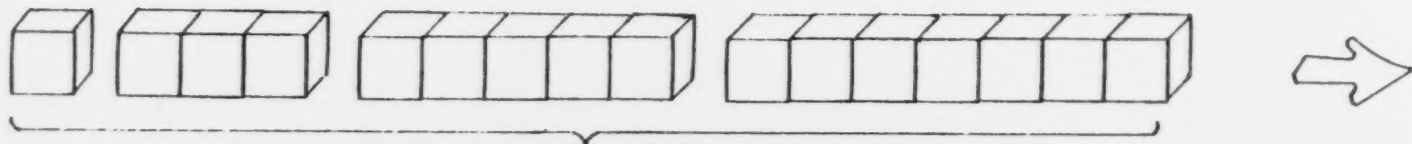
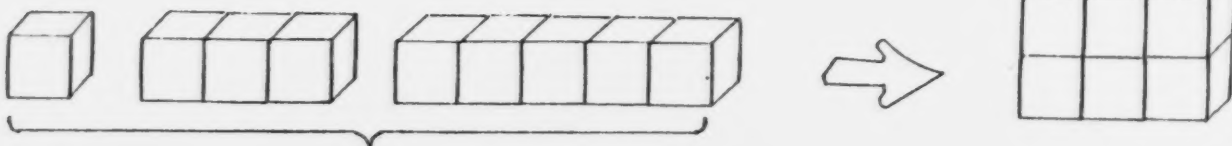
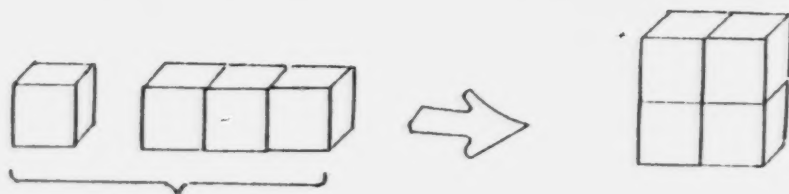
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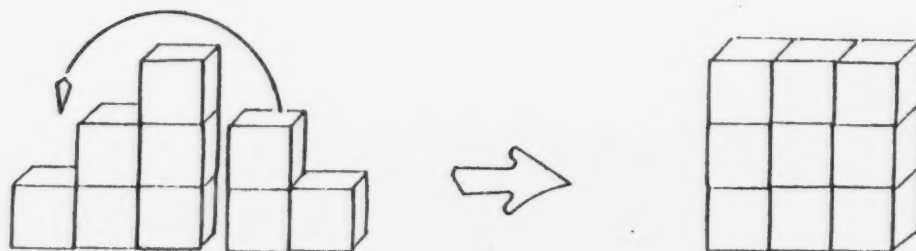
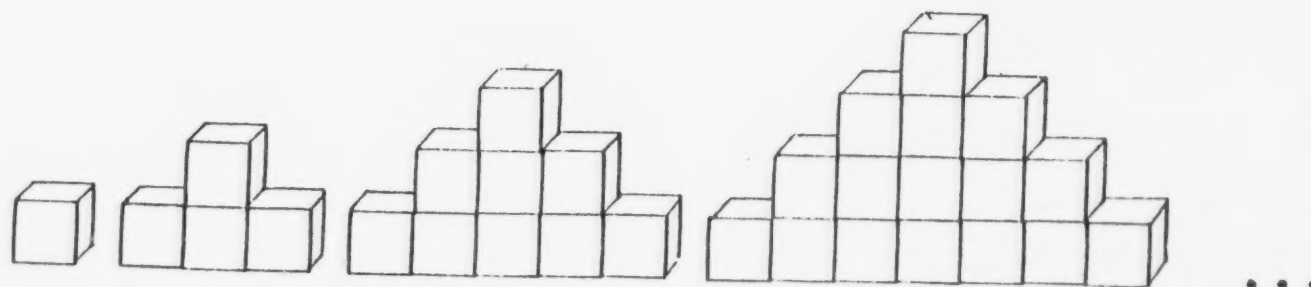
$$2n \quad n=1,2,3,4,\dots$$



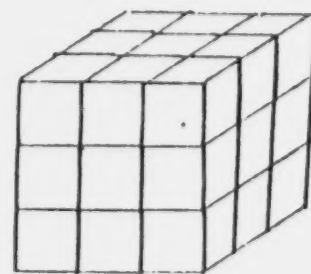
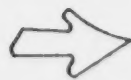
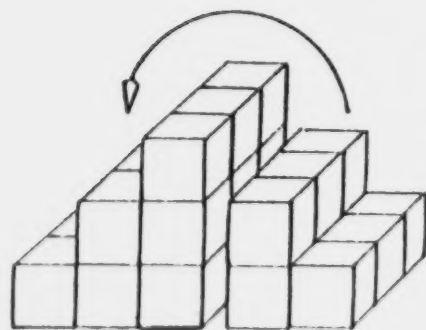
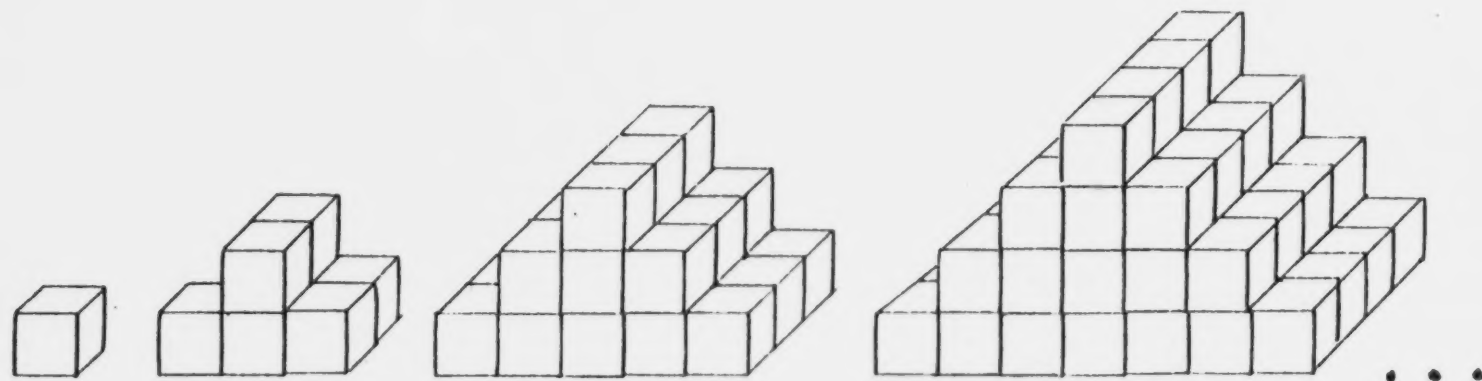
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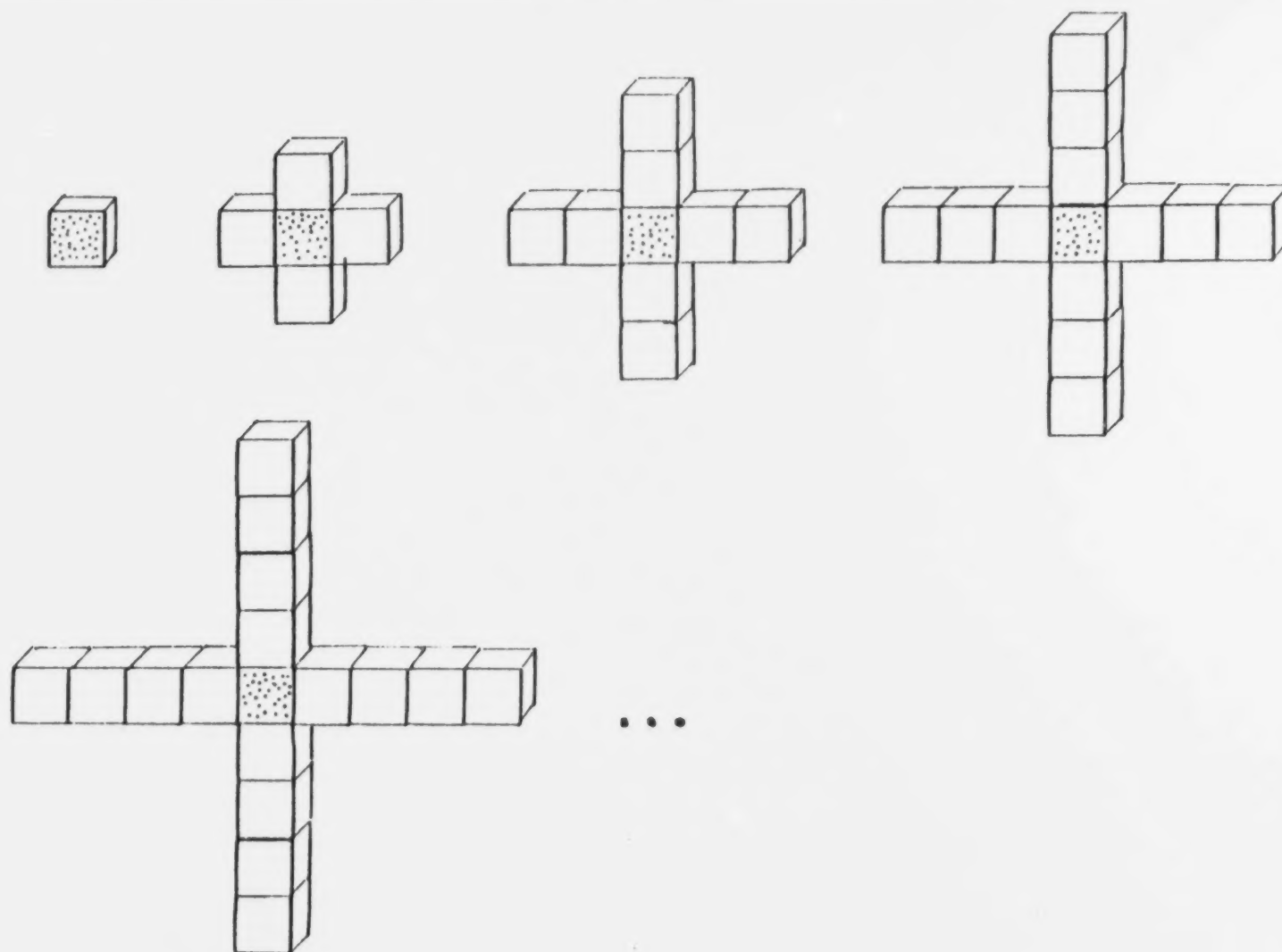
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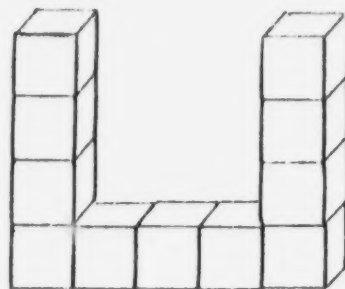
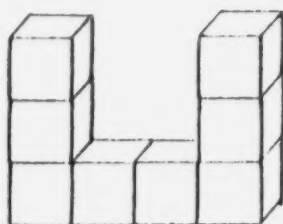
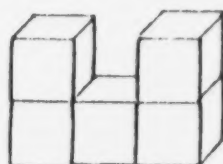
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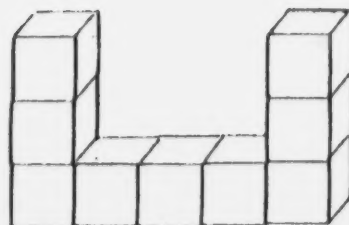
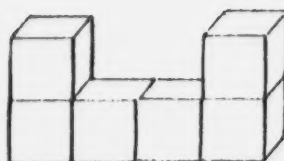
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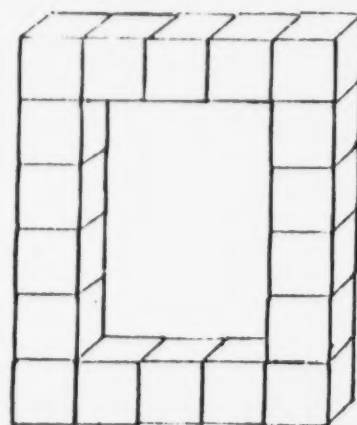
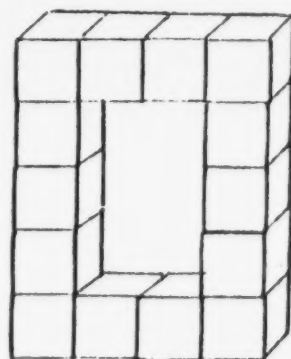
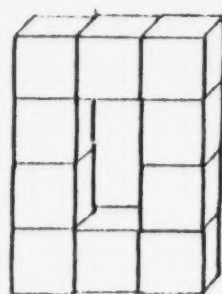


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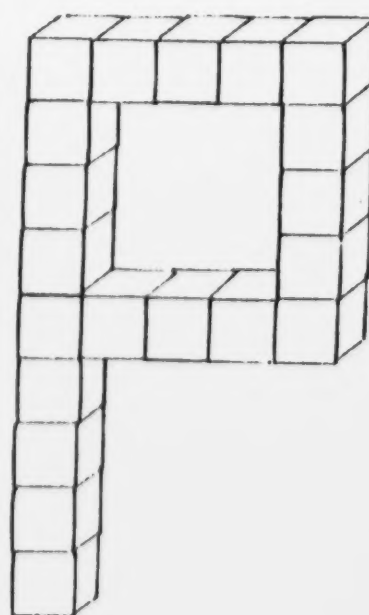
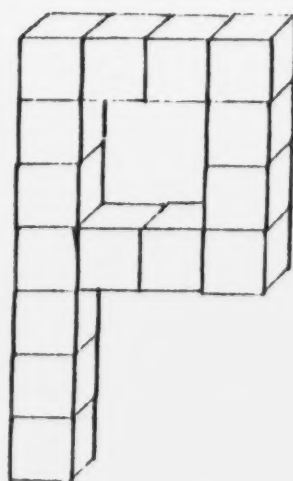
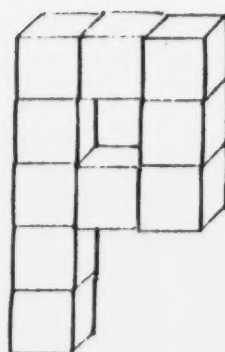


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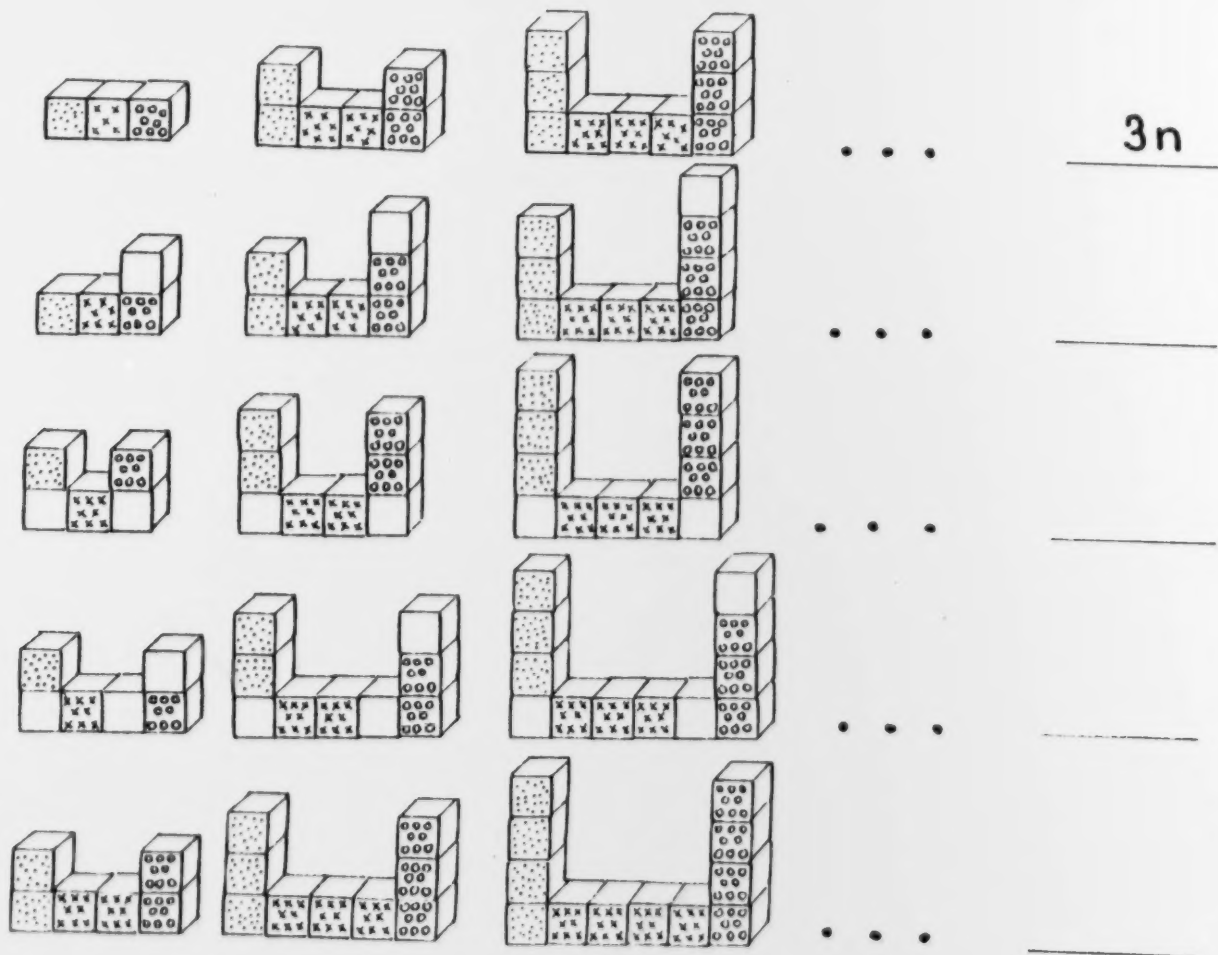
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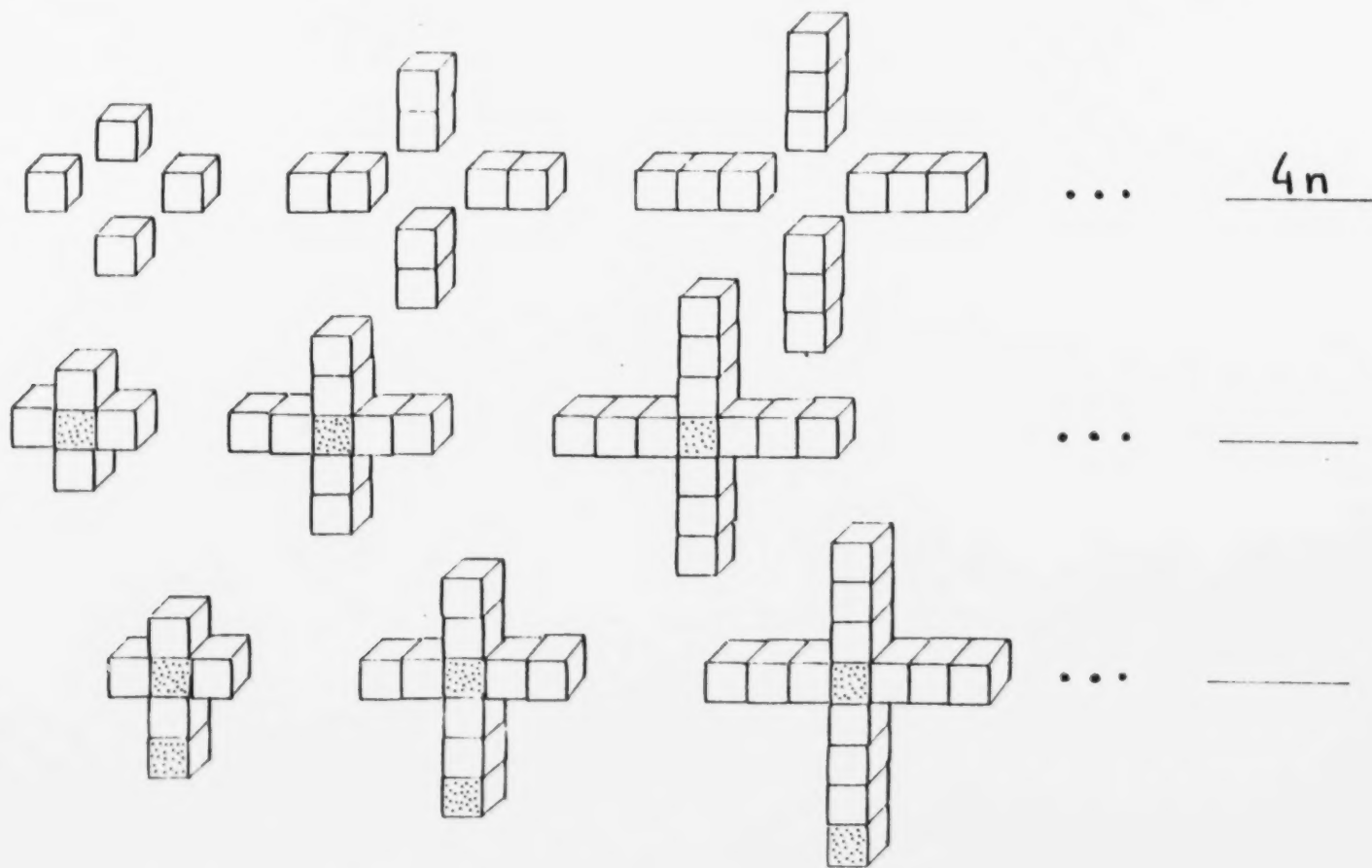
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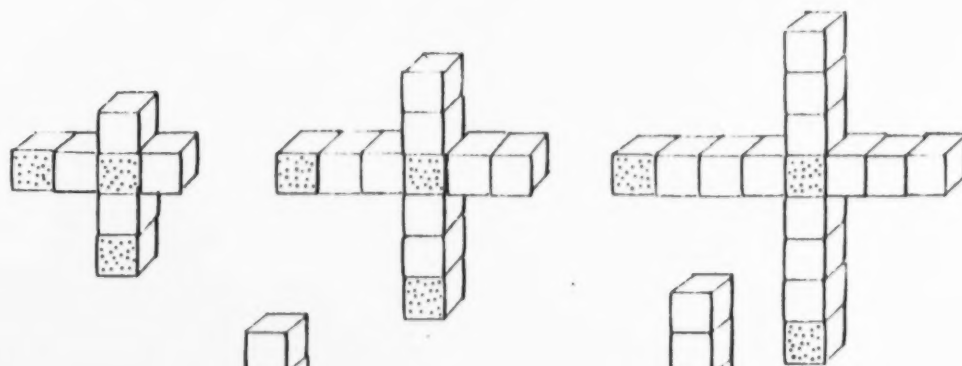
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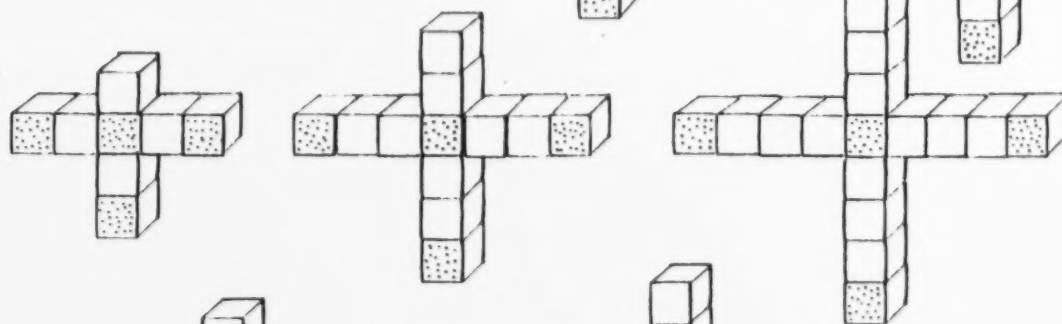
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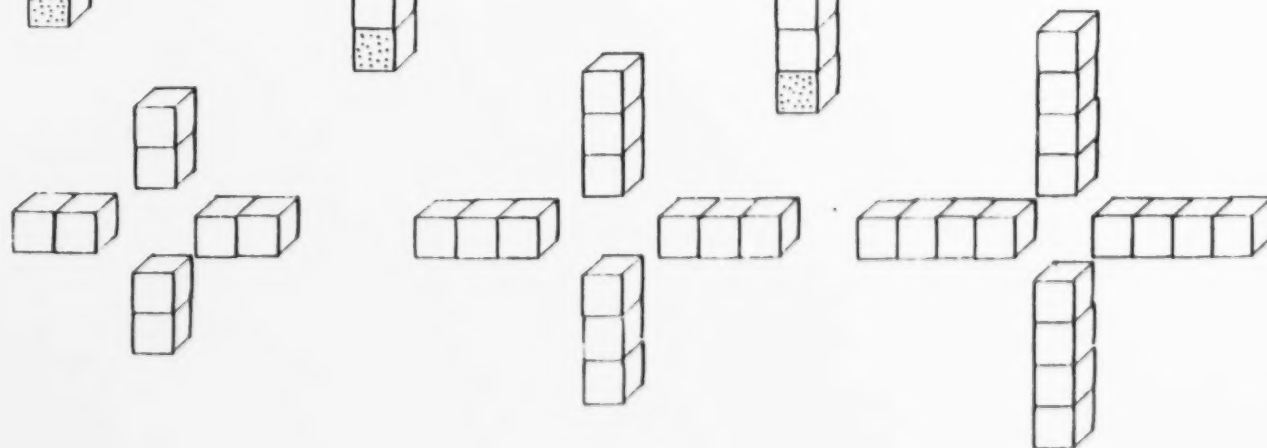
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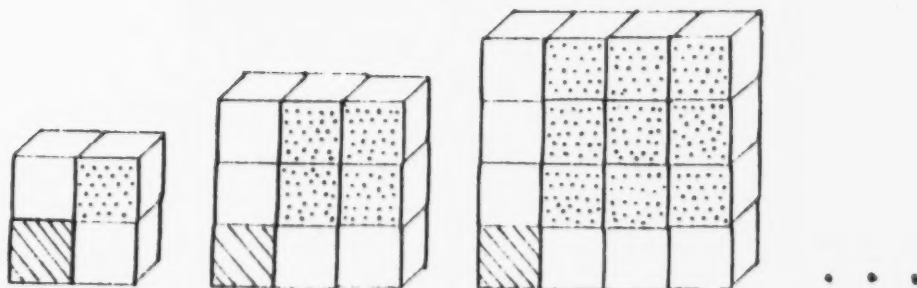
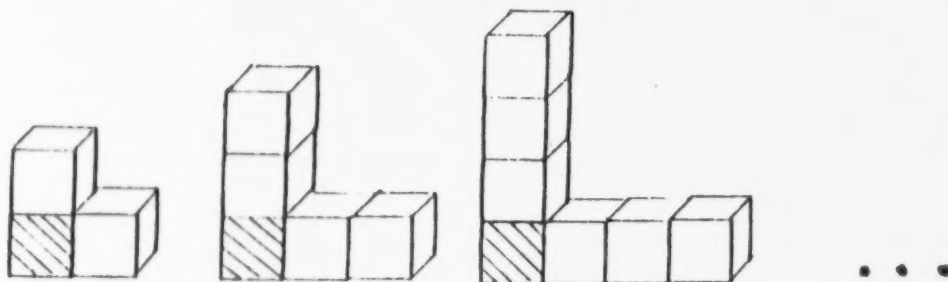


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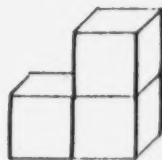


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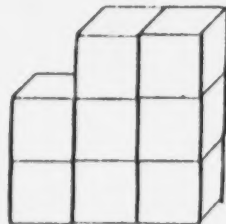
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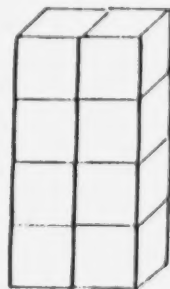
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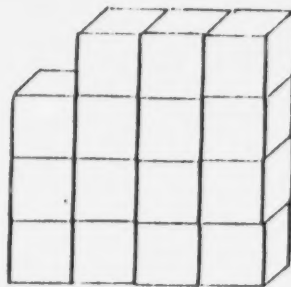
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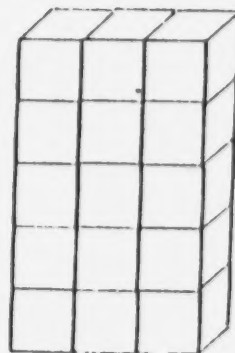
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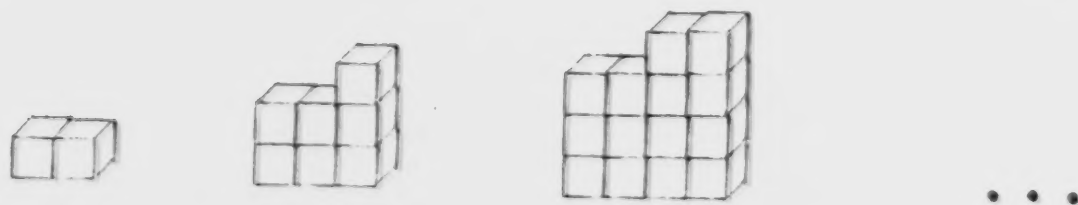
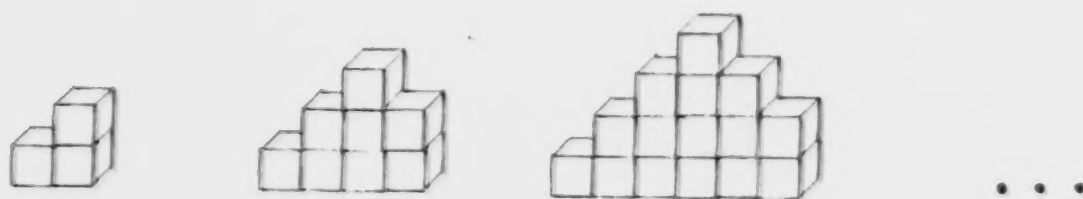
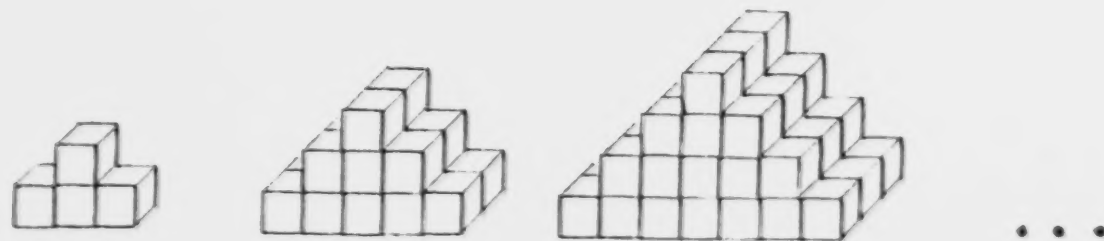


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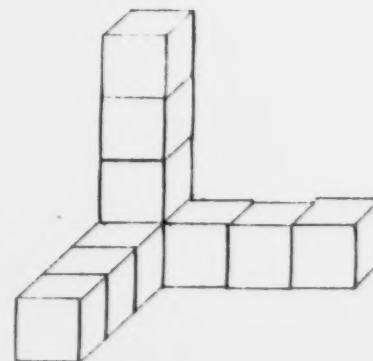
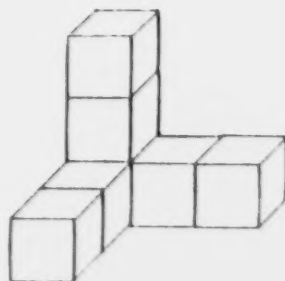
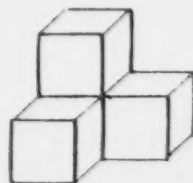
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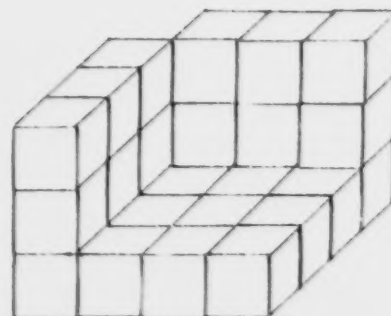
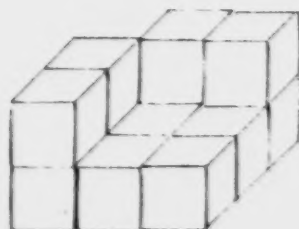
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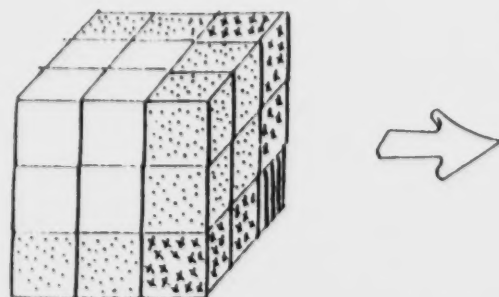
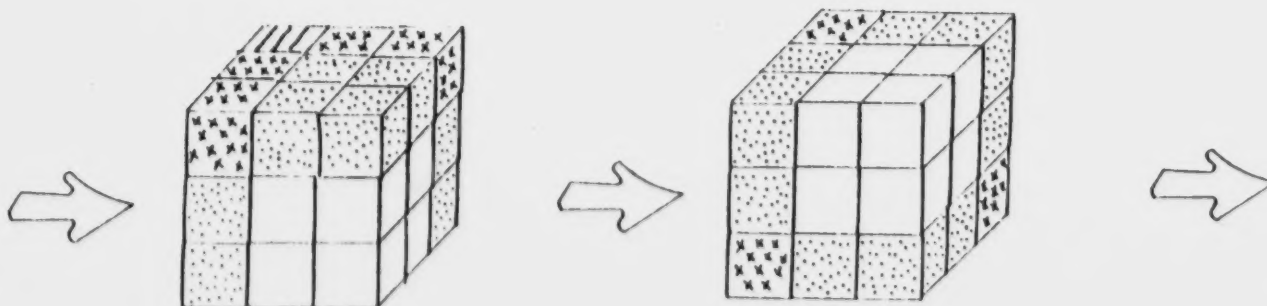
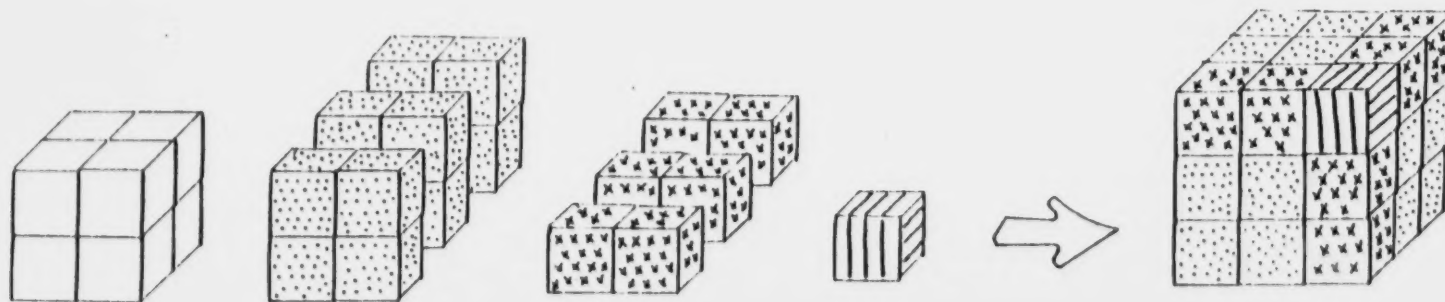


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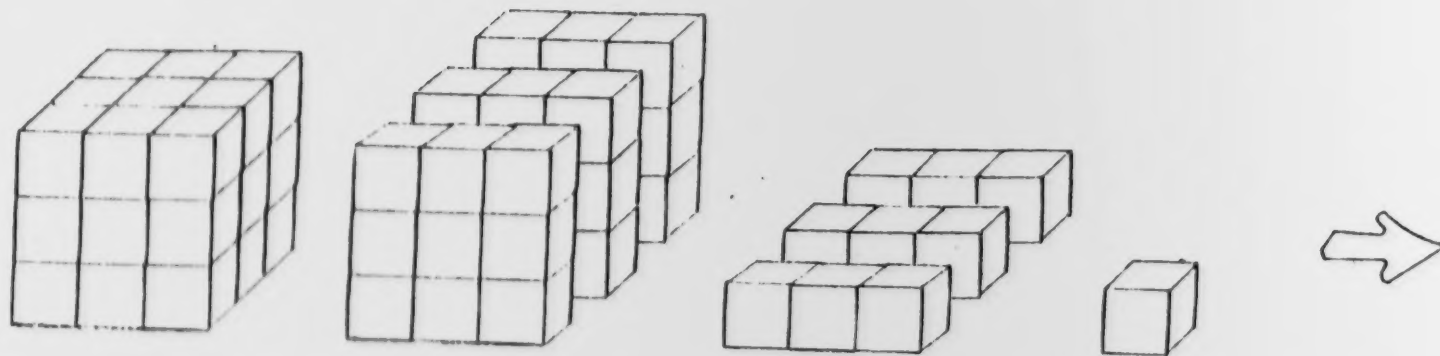
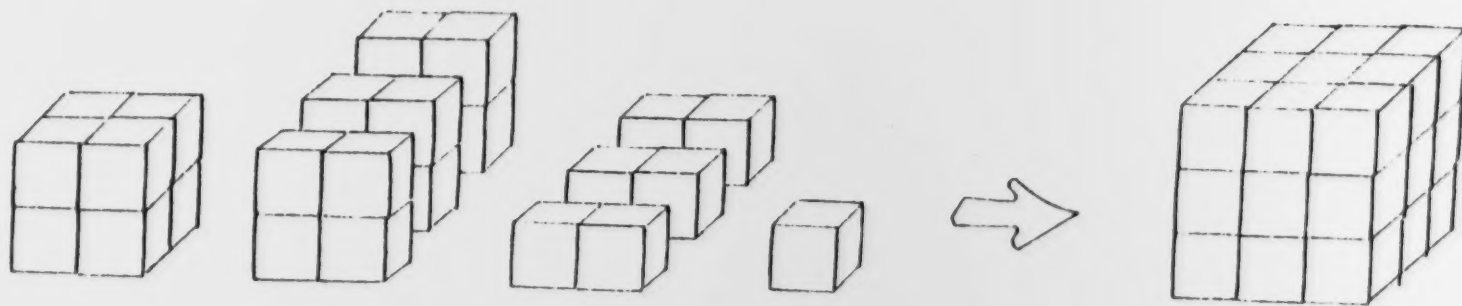


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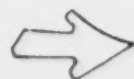
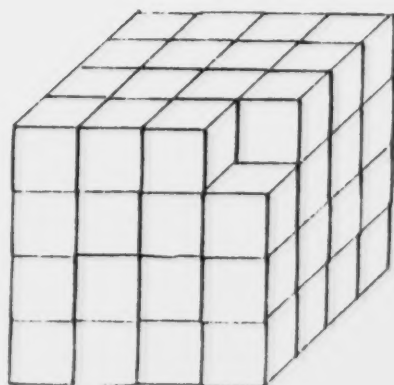
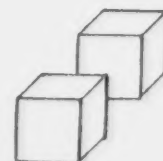
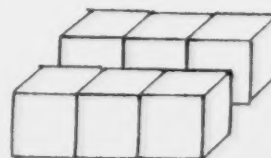
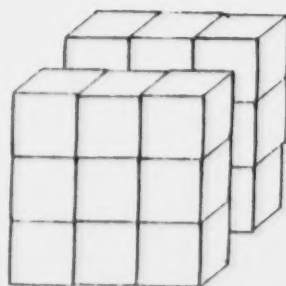
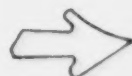
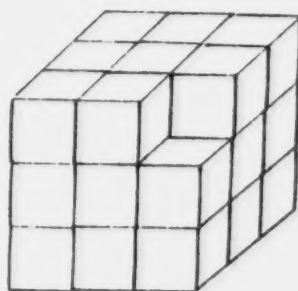
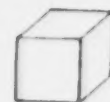
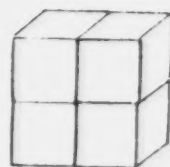
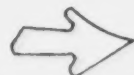
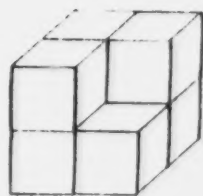
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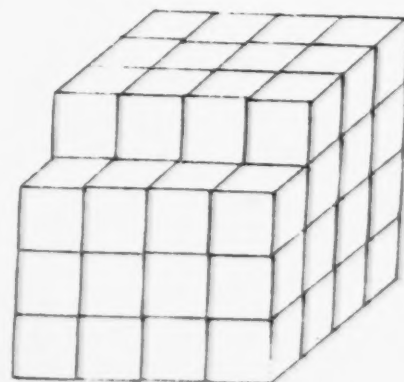
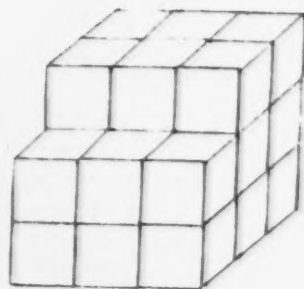
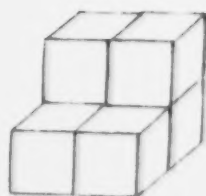
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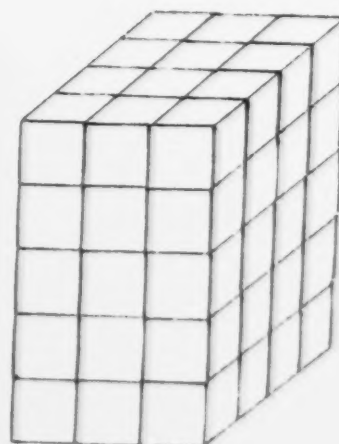
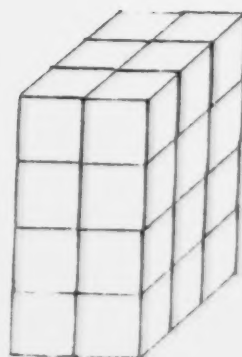
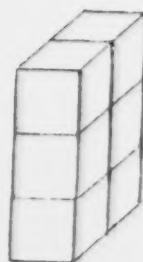
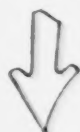
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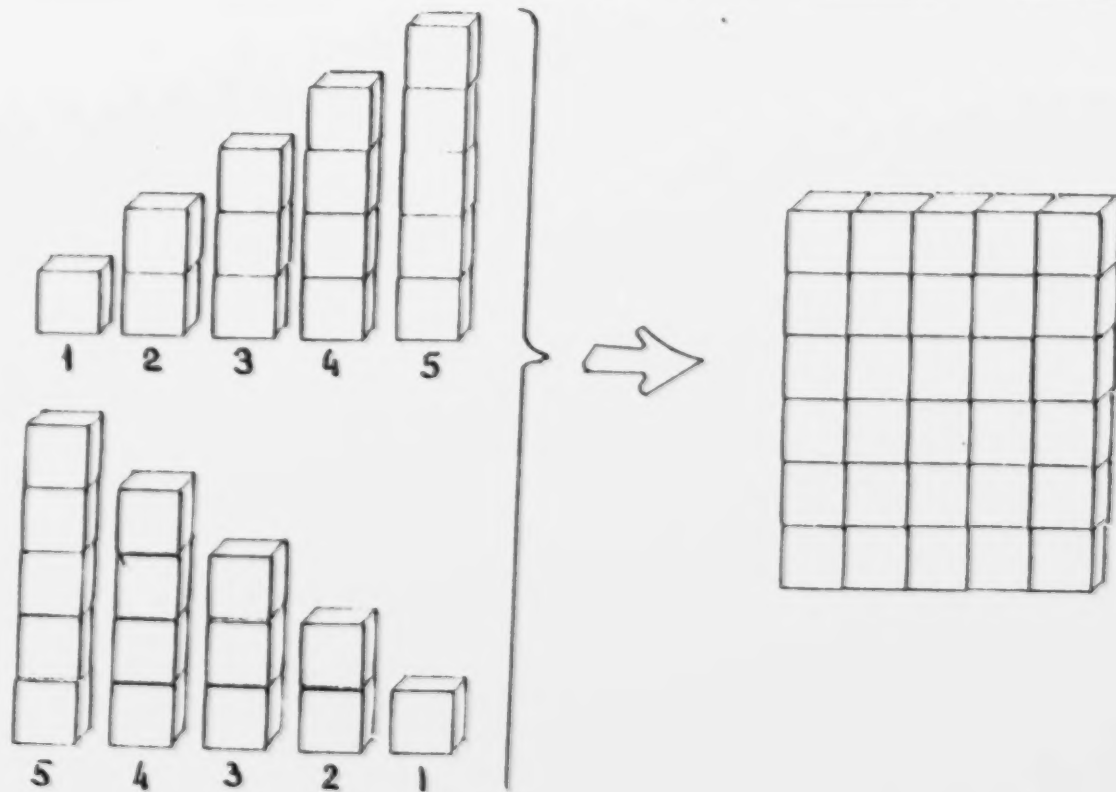
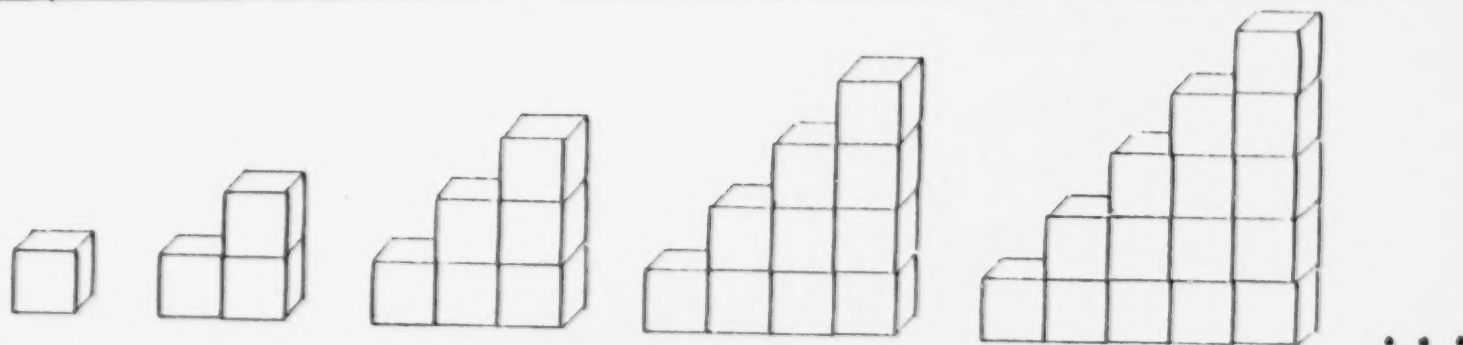


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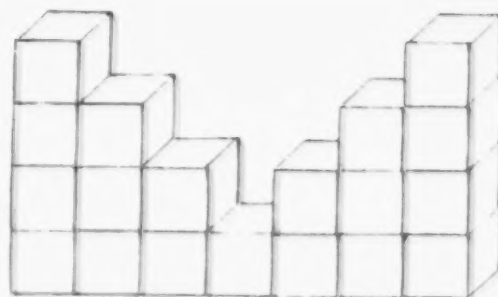
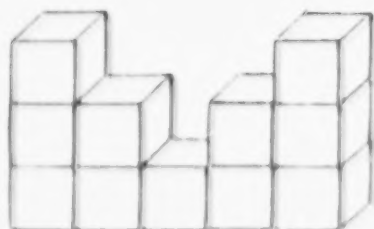
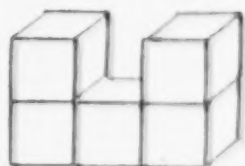


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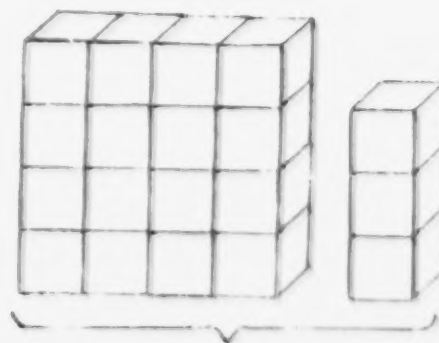
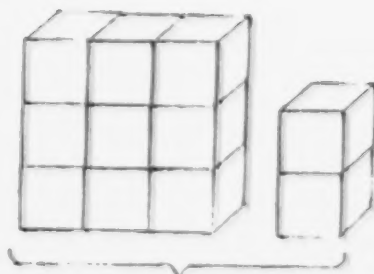
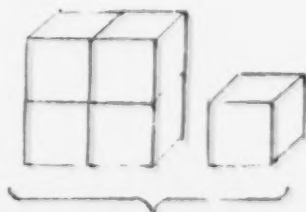
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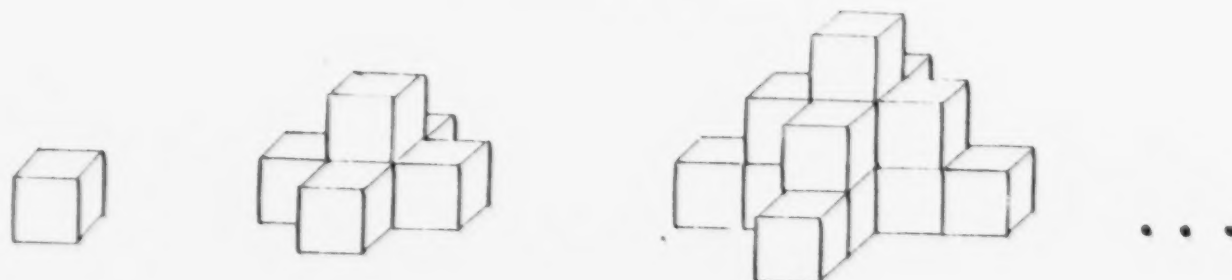
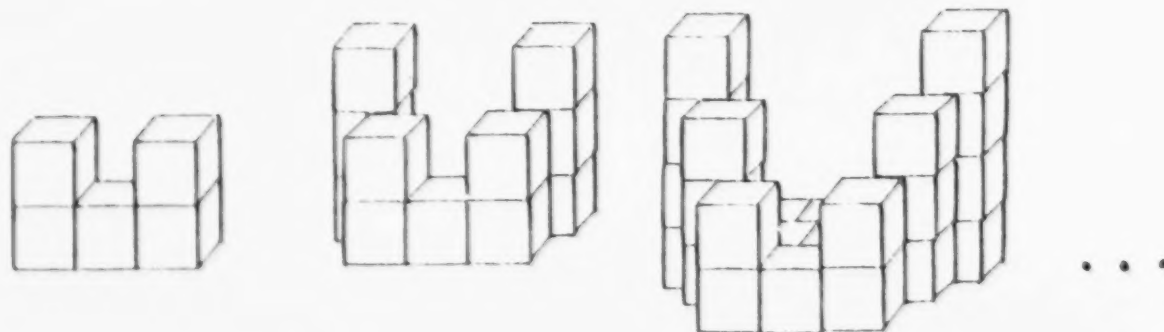


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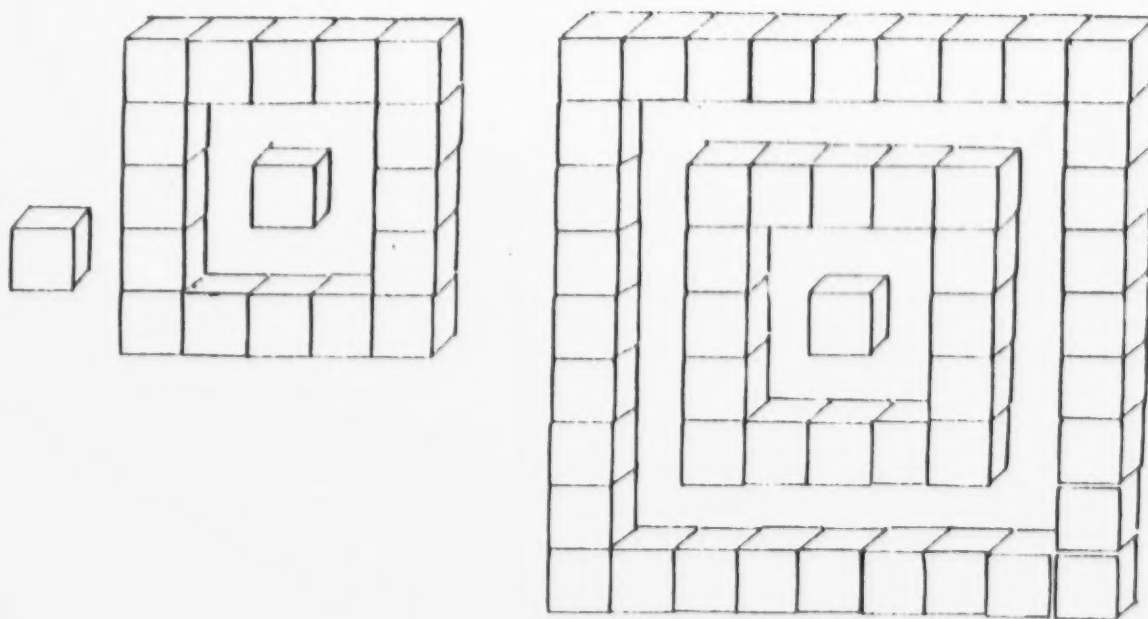


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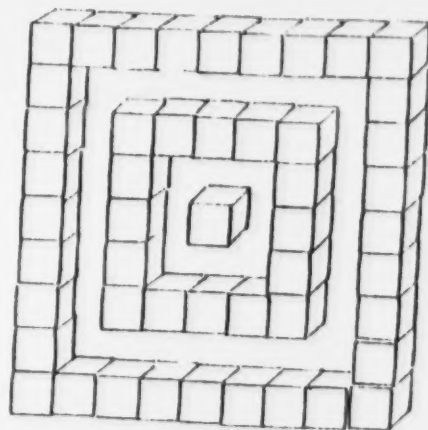
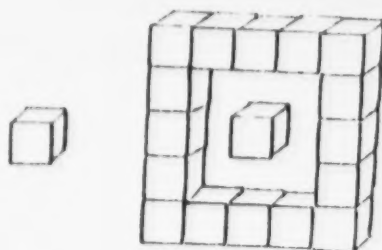
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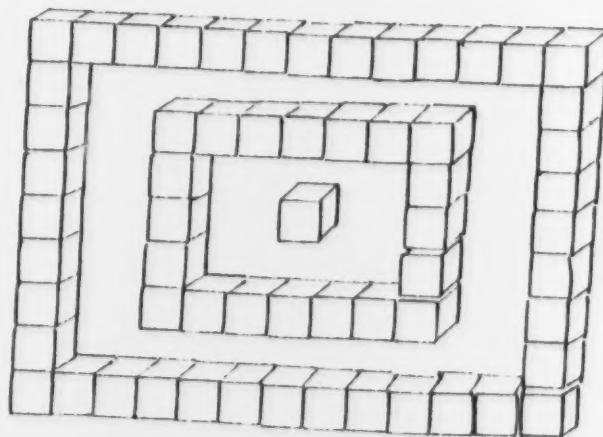
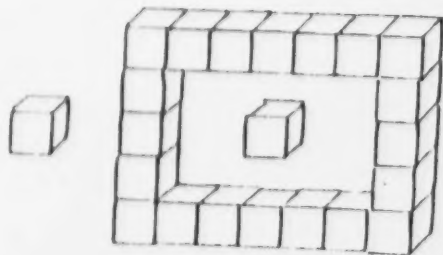
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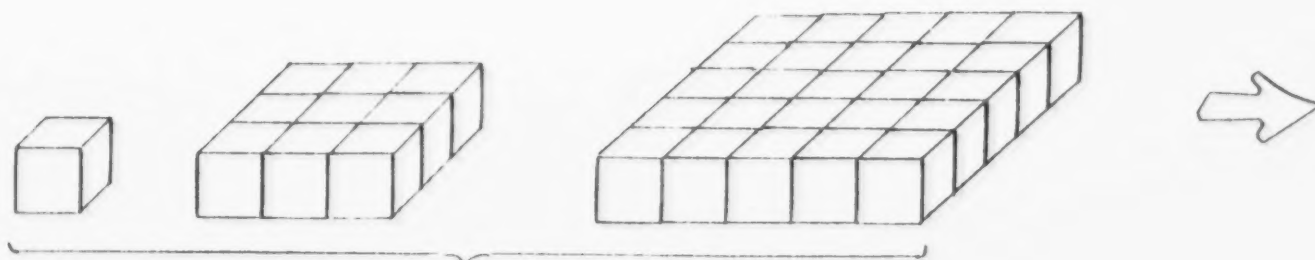
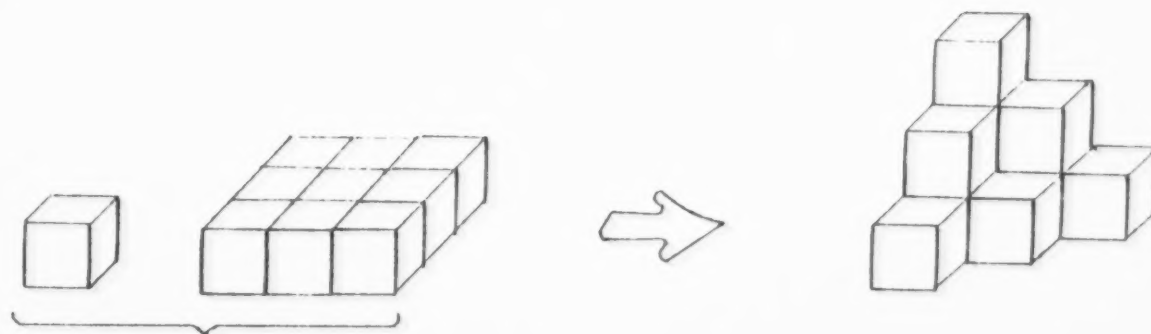
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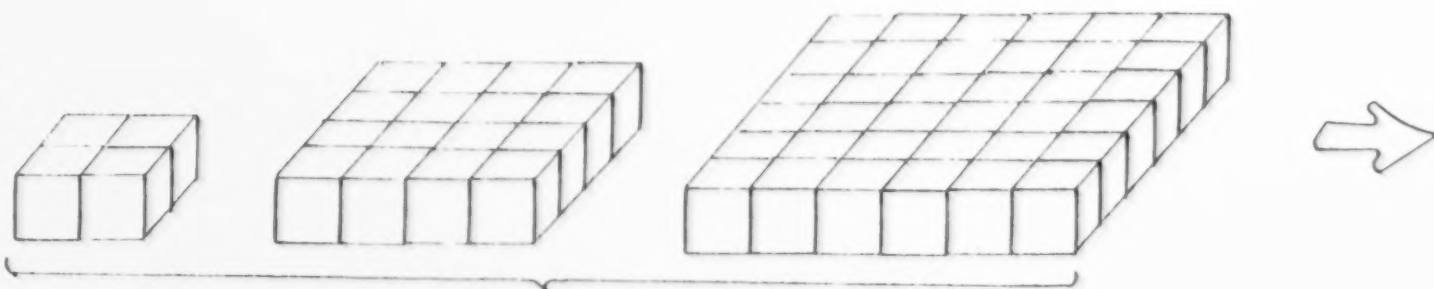
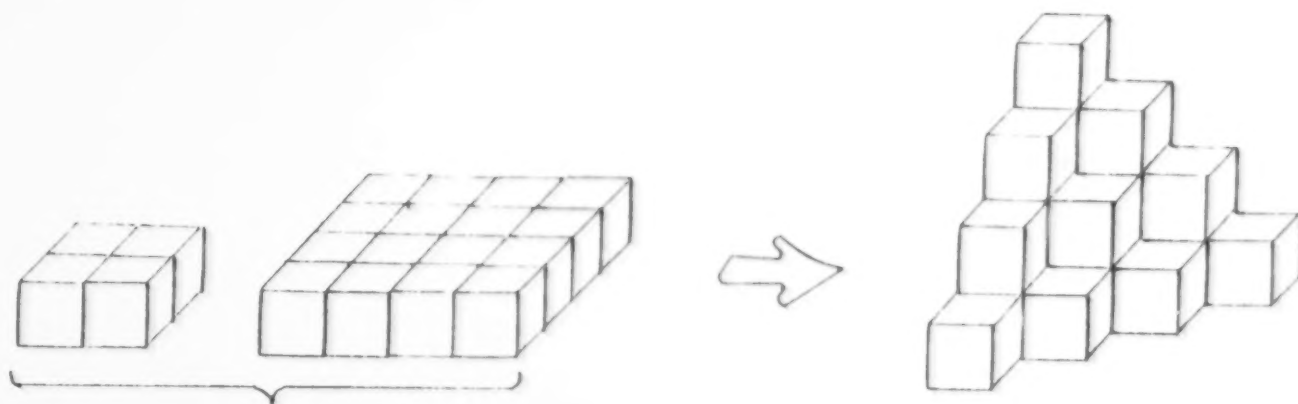
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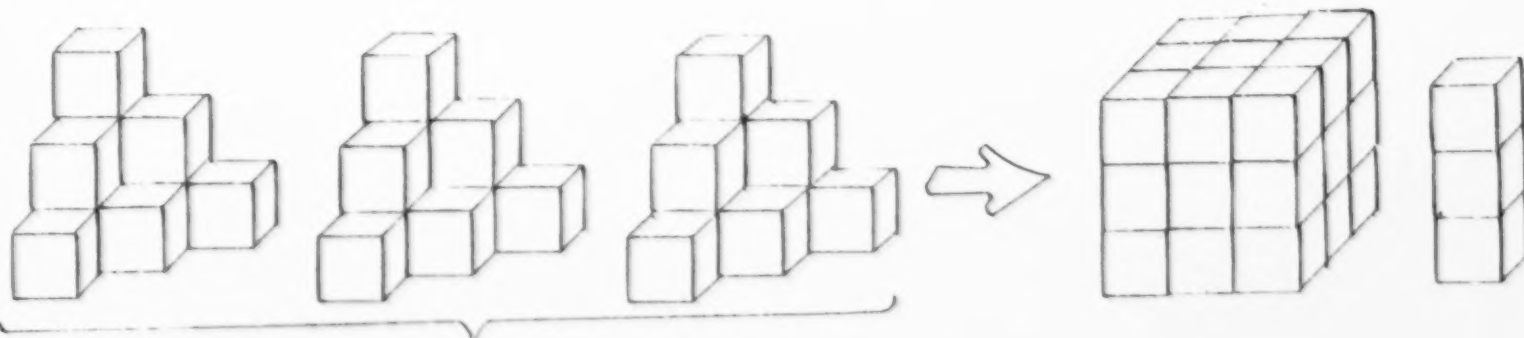
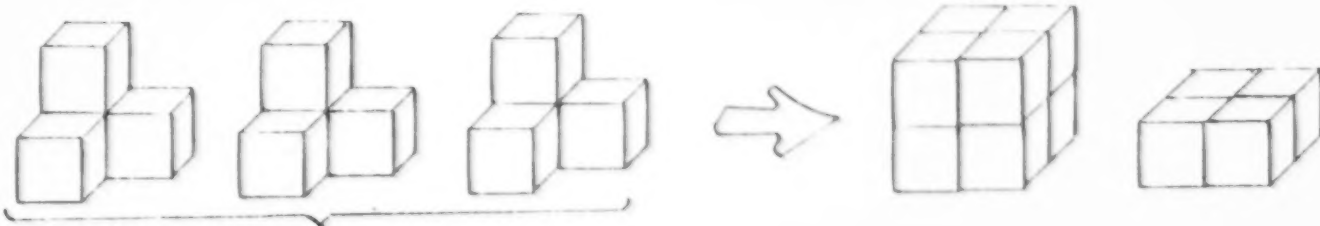
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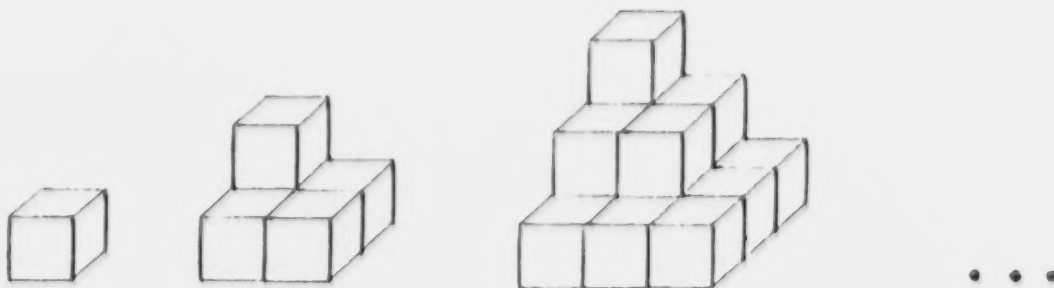
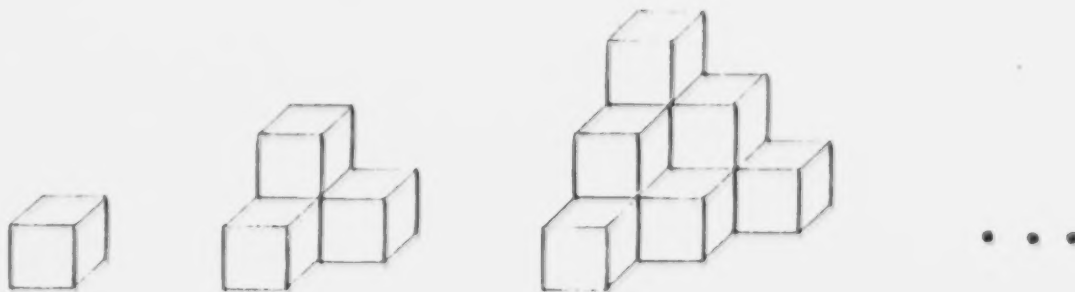


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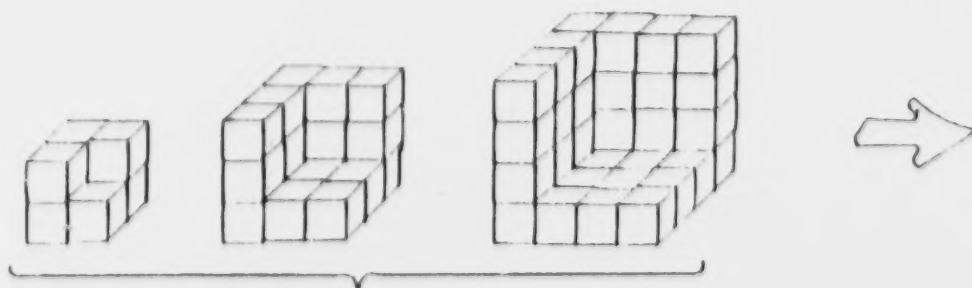
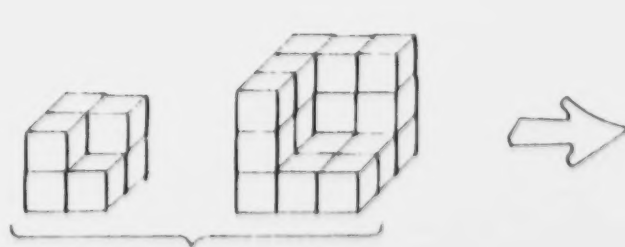
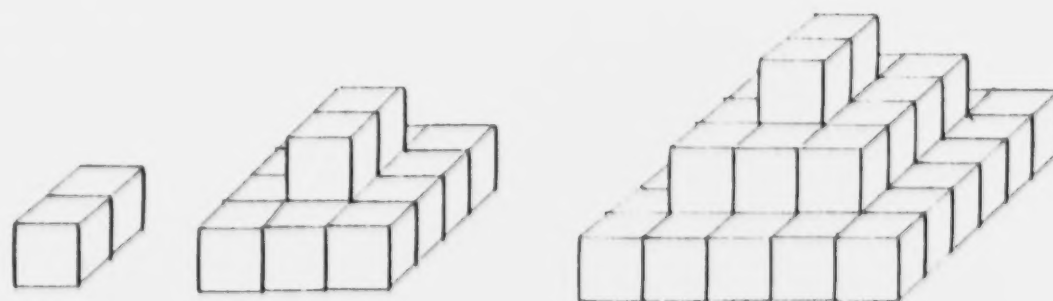


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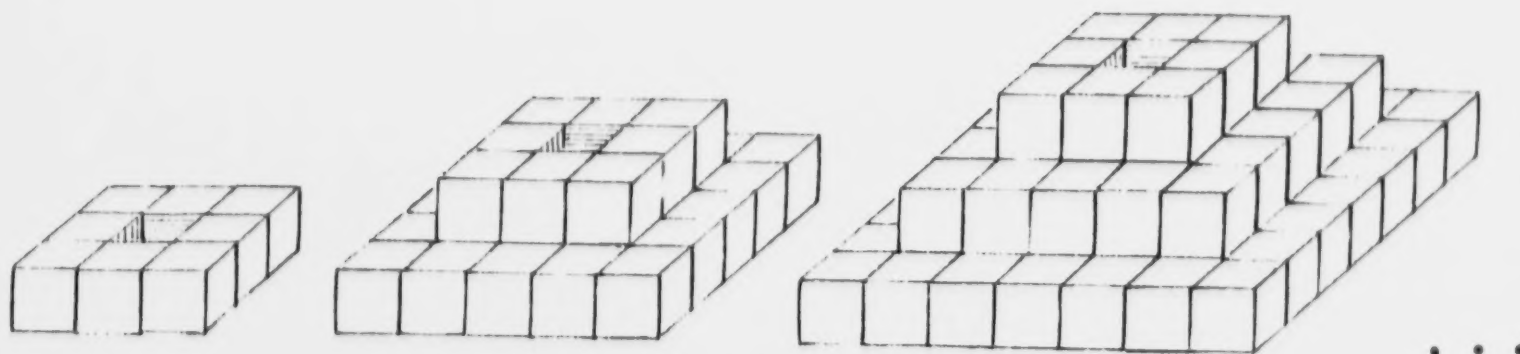
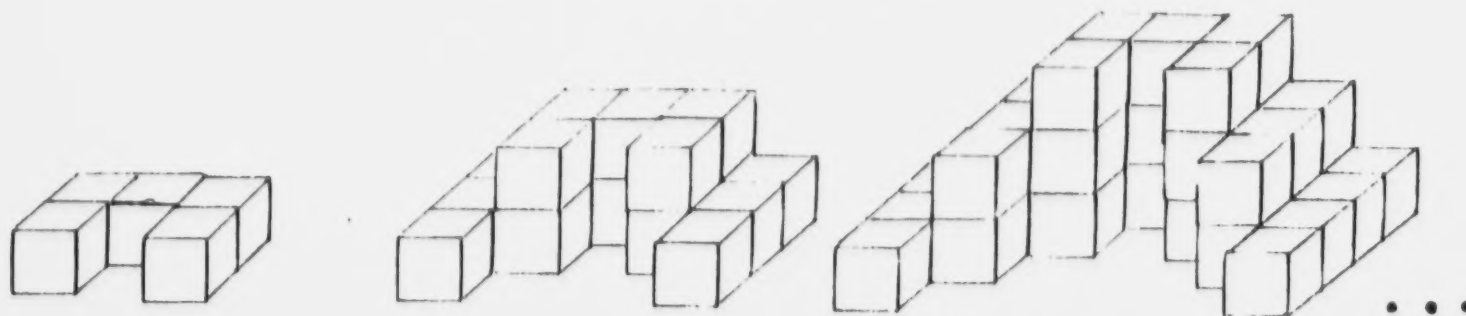


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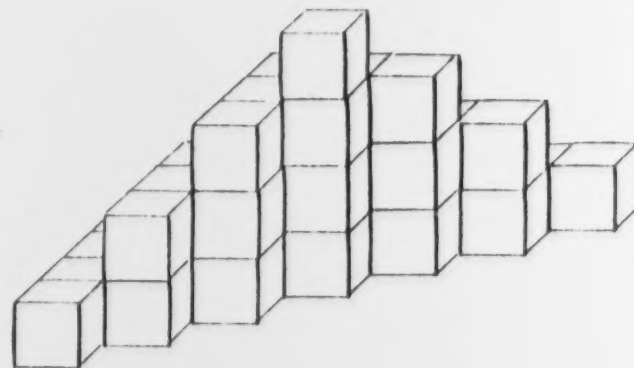
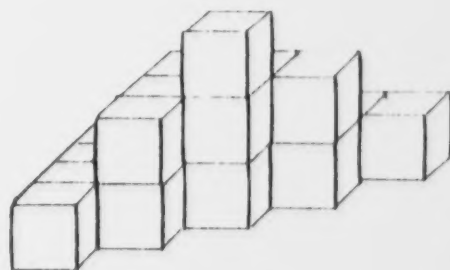
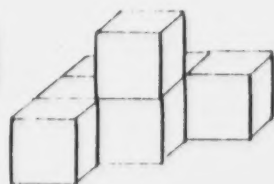


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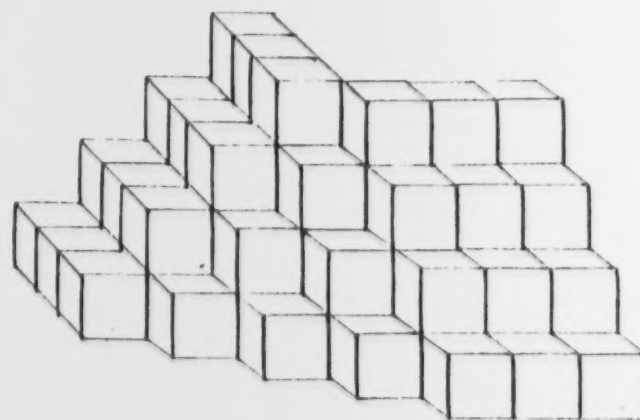
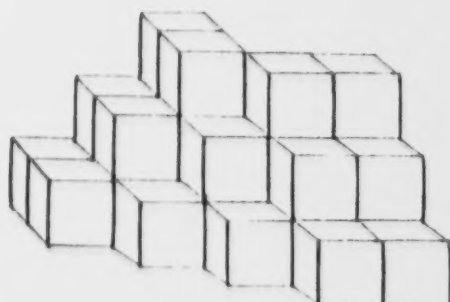
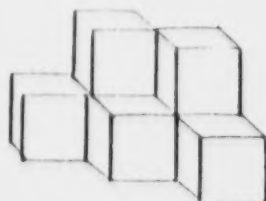
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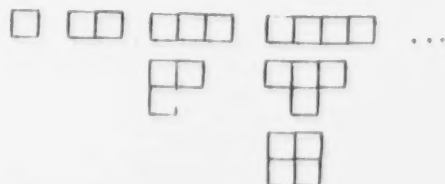


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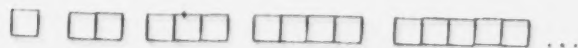
1.

Even numbers and odd numbers.

1. Build constructions which can represent numbers.



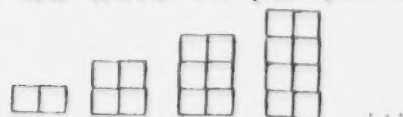
2. Organize your construction in a line and make an order, like 1, 2, 3, 4, 5, ...



3. Make two rows: even and odd numbers.



4. Place the row which represents even numbers in front of you. Rebuild each number in this way that you will have towers. The first floor of each tower has 2 blocks.



- How high is each tower? How many floors does it have?
- How many blocks has each 1st floor?
- How many blocks has each tower?
- Multiply number of blocks in 1st floor and number of floors: 2×1 , 2×2 , 2×3 , 2×4 , ...
- What is the result of multiplying?
- What is the relationship between the number of blocks for each tower and the height of each tower?

5. If I write an abbreviation for the height of a tower as "n", how can I write the abbreviation for the number of blocks in each tower?
What does the formula " $2n$ " show?

6. How many blocks do I have?

height 5 floor —
— " — 7 — " —
— " — 10 — " —
— " — 100 — " —

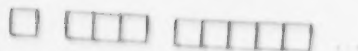
7. Check if your row of towers is in order and rebuild the odd numbered row similar to the even numbered row. Slide each odd numbered tower to the even numbered tower and compare each tower.



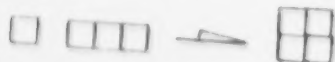
- What is the height of even and odd numbered towers?
- How many blocks does the odd numbered tower have?
- How can we write the abbreviation for the odd numbered towers?
Does this formula " $2n - 1$ " work?
- How many blocks do I have?

height 5 floor —
height 7 floor —
— " — 10 floor —
— " — 100 floor —

2. 1. Make a row of odd numbers

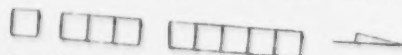


2. If I use only two odd numbers, is it possible to build a square?



Two odd numbers $\rightarrow$ the edge of the square is two!

3. Make a row of three odd numbers:



Is it possible to build a square? What is the edge of this square?

4. Use four, five, six, ... odd numbers...
- How many blocks has each edge of a square?
 - How many blocks has each square?
5. Build a square 5×5 . How many blocks do you need for this construction? Rebuild the square and find all odd numbers which make a sequence: 1, 2, ...

3. 1. Construct a sequence of stairs. How many blocks do you need for each next construction?
2. Rebuild each stair forming a square.
3. What is the abbreviation for each stair? Does the formula " n^2 " work?
4. Which stair do you have, when $n = 1, 2, \dots$

4. 1. Construct the double-wide stairs.
- How high is each next construction?
 - How wide is each next construction?
2. Rebuild each stair forming a cube.
3. Does the formula " n^3 " work?
4. What stair do you have when $n = 1, 2, \dots$

5. 1. What is the next "cross"-construction?
2. How many blocks has the arm of the next cross?
3. How many blocks do the following crosses have?
4. Does the formula " $4n + 1$ " work?

6. a) $3n + 2$; $n = 1, 2, \dots$
- b) $3n$; $n = 1, 2, \dots$

7. a) $4n - 2$; $n = 3, 4, \dots$ or $4n + 6$; $n = 1, 2, \dots$
- b) $5n$; $n = 2, 3, \dots$ or $5n + 5$; $n = 1, 2, \dots$

8. a) $3n$; $n = 1, 2, \dots$
- b) $3n + 1$; $n = 1, 2, \dots$
- c) $3n + 2$; $n = 1, 2, \dots$
- d) $3n + 3$; $n = 1, 2, \dots$
- e) $3(n + 1)$; $n = 1, 2, \dots$ } $3n + 3 = 3(n + 1)$

9. a) $4n$; $n = 1, 2, \dots$
- b) $4n + 1$; $n = 1, 2, \dots$
- c) $4n + 2$; $n = 1, 2, \dots$

10. a) $4n+3$; $n=1,2,\dots$
 b) $4n+4$; $n=1,2,\dots$
 c) $4(n+1)$; $n=1,2,\dots$ } $4n+4 = 4(n+1)$; $n=1,2,\dots$

11. a) $2n+1$; $n=1,2,\dots$
 b) $(n+1)^2 = n^2 + 2n + 1$; $n=1,2,\dots$

12. $n^2 - 1 = (n-1)(n+1)$; $n=1,2,\dots$

13. a) $n^3 - n^2 = n^2(n-1)$; $n=2,3,\dots$
 b) $n^3 - 1$; $n=2,3,\dots$
 c) $n^3 - 2$; $n=2,3,\dots$

14. a) $3n+1$; $n=1,2,\dots$
 b) $3n^2$; $n=1,2,\dots$

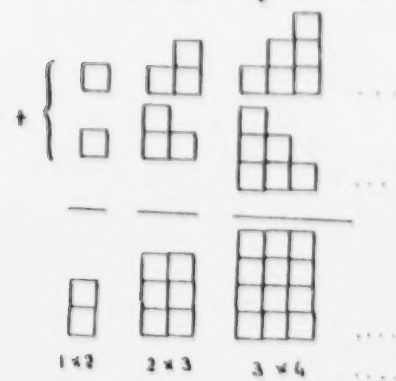
15. $(n+1)^3 = (n+1)(n+1)(n+1) = (n+1)(n^2 + 2n + 1)$; $n=1,2,\dots$

16. $(n+1)^3 = n^3 + 3n^2 + 3n + 1$; $n=1,2,\dots$

17. $n^3 - 1 = (n-1)(n^2 + n + 1)$; $n=2,3,\dots$

18. $n^3 - n = n(n-1)(n+1)$; $n=2,3,\dots$

19. 1. Build a single stair.
 2. Make a sequence of single stairs.
 3. Build a sequence of single stairs twice.
 4. Order each sequence in a different direction.



5. For each next stair I need $\frac{n(n+1)}{2}$ blocks!

6. a) $1+2+3+4 = ?$; $n=4$; $\frac{4 \times 5}{2} = 10$
 b) $1+2+\dots+20 = ?$; $n=20$; $\frac{20 \times 21}{2} = 210$

7. a) $\sum_{k=1}^n k = 10$

b) $\sum_{k=1}^{10} k = 210$

8. $\sum_{k=1}^n k = \frac{n(n+1)}{2}$

20. $n^2 + n - 1$; $n=2,3,\dots$

$$21. \quad a) \quad \sum_{k=1}^n 3k+2 = \sum_{k=1}^n 3k + \sum_{k=1}^n 2 = 3 \sum_{k=1}^n k + 2 \sum_{k=1}^n 1 =$$

$$= \frac{3}{2} n(n+1) + 2n = \frac{n}{2} (3n+3+4) = \frac{n}{2} (3n+7); n=1, 2, \dots$$

$$b) \quad 1 + \sum_{k=1}^n 4n+1 = 1 + 4 \sum_{k=1}^n n + \sum_{k=1}^n 1 = 1 + 2n(n+1) + n =$$

$$= 2n^2 + 3n + 1 = n(2n+3) + 1; \quad n=0, 1, 2, \dots$$

$$22. \quad 1 + \sum_{k=1}^n 16k = 1 + 16 \sum_{k=1}^n k = 1 + 8n(n+1) =$$

$$= 8n^2 + 8n + 1; \quad n=0, 1, 2, \dots$$

$$23. \quad \begin{array}{ll} (2 \times 4)n(n+1) + 1 & n=0, 1, 2, \dots \\ (2 \times 5)n(n+1) + 1 & n=0, 1, 2, \dots \\ (2 \times 6)n(n+1) + 1 & \\ \dots & \\ 2(k+3)n(n+1) + 1 & n=0, 1, 2, \dots \\ & k=1, 2, \dots \end{array}$$

$$24. \quad \sum_{k=1}^n (2n-1)^2 \quad \text{--- odd tower} \quad \left(\frac{n}{3} (2n-1)(2n+1); n=1, 2 \right)$$

$$25. \quad \sum_{k=1}^n (2n)^2 \quad \text{--- even tower} \quad \left(\frac{n}{3} (4n^2 + 6n + 2); n=1, \dots \right)$$

$$26. \quad \begin{array}{ll} n=2 & 12 = 8 + 4 \\ n=3 & 30 = 27 + 3 \\ n=4 & 60 = 64 - 4 = 64 - (1 \times 4) \\ n=5 & 105 = 125 - 20 = 125 - (4 \times 5) \\ n=6 & 162 = 216 - 54 = 216 - (8 \times 6) \\ n=7 & 252 = 343 - 91 = 343 - (13 \times 7) \\ n=8 & \quad \quad \quad - (19 \times 8) \\ n=9 & \quad \quad \quad - (26 \times 9) \end{array}$$

$$n^3 - \frac{n(n+1)}{2} (n+3) =$$

$$\left. \begin{array}{l} \frac{(n^2+n)(n+3)}{2} = \frac{n^3+4n^2+3n}{2} \\ \frac{(n^3+n)(n+4)}{2} = \frac{n^3+5n^2+4n}{2} \end{array} \right\} \quad \text{--- } \frac{n^2+n}{2}$$

$$\frac{1}{2} (n^3+4n^2+3n - n^3-n) = (n^2+3n+2) \frac{1}{2}$$

$$\frac{n}{6} (n^2+3n+2); \quad n=1, 2, \dots$$

$$\text{for } n=2n-1; \quad \frac{n}{3} (2n-1)(2n+1) \quad n=1, 2, \dots$$

$$n=2n; \quad \frac{n}{3} (4n^2+6n+2) \quad n=1, 2, \dots$$

$$27. \quad a) \quad \sum_{k=1}^n \frac{n(k+1)}{2} = \frac{n}{6} (n^2+3n+2); \quad n=1, 2, \dots$$

$$b) \quad \sum_{k=1}^n k^2 = \frac{n}{6} (4n^2+3n+2); \quad n=1, 2, \dots$$

$$28. \quad a) \quad \sum_{k=1}^n (2k-1)2k = 4 \sum_{k=1}^n k^2 - 2 \sum_{k=1}^n k = \frac{4n}{6} (2n^2+3n+1) - n(n+1) =$$

$$= \frac{n}{3} (4n^2+3n-1) \quad n=1, 2, \dots$$

$$b) \quad n^3 - 1; \quad n=2, 3, \dots$$

$$29. \quad a) \quad \sum_{k=1}^n k^2 + k - 1 = \sum_{k=1}^n (k+1)^2 + (k+1) - 1 = \frac{n}{6} (2n^2+12n+16) \quad n=1, 2, \dots$$

$$b) \quad \sum_{k=1}^n (2n+1)^2 - 1 = \frac{n}{3} (4n^2+3n+2) \quad n=1, 2, \dots$$

$$30. \quad a) \quad \sum_{k=1}^n (k+1)(2k+1) = \sum_{k=1}^n 2k^2+3k+1 = \frac{2}{3} [4n^2+15n+17] + 1 \quad n=1, 2, \dots$$

$$b) \quad \sum_{k=1}^n \frac{k(k+1)}{2} + 2n \sum_{k=1}^n k+1 = \frac{n}{6} (n^2+3n+2) + n(n+1)(n+2) =$$

$$= \frac{n}{6} (7n^2+21n+14) \quad n=1, 2, 3, \dots$$